Eur. Phys. J. B **29**, 233–236 (2002)
DOI: 10.1140/epjb/e2002-00292-8

THE EUROPEAN PHYSICAL JOURNAL B

Screw photon-like (3+1)-solitons in extended electrodynamics

S.G. Donev[a]

Institute for Nuclear Research and Nuclear Energy, Bulg. Acad. Sci., 1784 Sofia, blvd.Tzarigradsko chaussee 72, Bulgaria

Received 27 September 2001 / Received in final form 18 March 2002
Published online 2 October 2002 –

Abstract. This paper presents photon-like spatially finite soliton solutions of screw type to the vacuum field equations of Extended Electrodynamics (EED) in relativistic formulation. Also, a new description of the spin momentum inside EED, based on the notion for energy-momentum exchange between F and $*F$, is introduced and used to compute the integral spin momentum of a screw soliton. The Planck's formula $E = h\nu$ in the form of $ET = h$ arises as a measure of the integral spin momentum.

PACS. 11.10.Lm Nonlinear or nonlocal theories and models – 11.27.+d Extended classical solutions; cosmic strings, domain walls, texture

1 Introduction

Natural objects may be classified according to various principles. The classical point-like objects (called usually particles) are allowed to interact continuously with each other just through exchanging (through some mediator usually called field) universal conserved quantities: energy, momentum, angular momentum, so that, the set of objects "before" interaction is the same as the set of objects "after" interaction, no objects have disappeared and no new objects have appeared, only the conserved quantities have been redistributed. This is in accordance with the assumption of point-likeness, *i.e.* particles are assumed to have no internal structure, so they are undestroyable. Clearly, there are no such objects in Nature, nevertheless, under *suitable conditions* the point-likeness hypothesis has proved to be useful. And these suitable conditions determine the frame where classical mechanics works well.

A basic feature of microobjects (called usually elementary particles: photons, electrons, etc.) is that a set of microobjects may transform into *another* set of microobjects under definite conditions, for example, the well known annihilation process: $(e^+, e^-) \rightarrow 2\gamma$. These transformations obey also the energy-momentum and angular+spin momentum conservation, but some features may disappear (*e.g.* the electric charge) and new features (*e.g.* motion with the highest velocity) may appear. Hence, microobjects *allow to be destroyed*, so they *have structure* and, consequently, the point-like approximation has to be reconsidered. Quantum Electrodynamics has shown how this reconsideration may be done.

Although the great successes of Quantum Electrodynamics, one would like to have a theory, where microobjects are not considered as point-like objects, and modern (super)string theory is a step towards building such a theory. Earlier views and attempts in this direction are also available [1].

On the other hand, it seems reasonably to examine different theoretical approaches to describe finite (or extended) 3d-objects with structure as time-dependent solutions to partial differential equations. If we find a self-consistent classical theory giving $3d$-soliton solutions which have integral properties recalling the properties of the known microobjects, in our view, such a theory worths being studied carefully.

This paper aims to present a class of solutions to a particular extension of Maxwell equations which we called Extended Electrodynamics (EED) [2]. These solutions have a *3d screw nature*, we call them *screw solitons*, and they are interesting in, that they show a photon-like behavior and properties. Determining the degree of adequacy of these solutions to the real photons, needs, of course, time and additional serious study. We think that having at hand such classical soliton solutions with properties close to those of photons may only help in our efforts to understand more deeply the quantum nature of microobjects.

We proceed now to recall the main features/properties of solitons and photons.

2 Solitons and photons

The concept of soliton appears in physics as a nonlinear elaboration – physical and mathematical – of the general notion for excitation in a medium. It includes the following features:

I. Physical

1. The medium is homogeneous, isotropic and has definite properties of elasticity.

2. The excitation does not destroy the medium.

[a] e-mail: sdonev@inrne.bas.bg

3. The excitation is finite:

– at every moment it occupies comparatively small volume of the medium;

– it carries finite quantities of energy-momentum and angular momentum, and of any other physical quantity;

– it may have translational-rotational (may time-periodical) dynamical structure, *i.e.* besides its straightline propagation as a whole it may have internal rotational degrees of freedom.

4. The excitation is time-stable, *i.e.* at lack of external perturbations its dynamical evolution does not lead to a self-ruin. In particular, the spatial shape of the excitation does not (significantly) change during its propagation.

The above 4 features outline the physical notion of a *solitary wave*. A solitary wave becomes a *soliton* if it has in addition the following property of stability:

5. The excitation survives when collides with another excitation of the same nature.

II. Mathematical

1. The excitation defining functions Φ^a are components of *one* mathematical object (usually a section of a vector/tensor bundle) and depend on n spatial and 1 time coordinates. This feature introduces some notion of *integrity*: *one excitation – one mathematical object*, although having many algebraically independent but differentially interrelated (through the equations) components Φ^a.

2. The components Φ^a satisfy some system of nonlinear partial differential equations (except the case of (1+1) linear wave equation), and represent some "running wave" dynamics as a whole, together with available internal dynamics.

3. There are (infinite) many conservation laws.

4. The components Φ^a are localized (or finite) functions with respect to the spatial coordinates, and the conservative quantities are finite.

5. The multisoliton solutions, describing elastic interaction (collision), tend to many single soliton solutions at $t \to \infty$.

The above physical/mathematical features are not always strictly accounted for in the literature. For example, the word *soliton* is frequently used for a *solitary wave* excitation. Another example is the usage of the word soliton just when the energy density, has the above soliton properties [3]. Also, one usually meets this soliton terminology for spatially *localized*, *i.e.* going to zero at spatial infinity, but *not* spatially *finite* Φ^a, *i.e.* when the spatial support of Φ^a is a compact set. In fact, all soliton solutions of the well known KdV, SG, NLS equations are localized and *not* finite.

Further in this paper we shall present 1-soliton screw solutions of the vacuum EED equations, so we shall use the word soliton for solitary wave. The screw soliton solutions we are going to present are of photon-like character, *i.e.* the velocity of their translational component of propagation is equal to the velocity of light c, and besides of the energy-momentum, they carry also spin momentum accounting for the available rotational component of propagation. We consider appropriate at this moment to recall some of the well known properties of *photons.*

1. Photons have zero proper mass and electric charge. The (straightline) translational component of their propagation velocity is constant and equal to the experimentally established velocity of light in vacuum.

2. Photons are time-stable objects. Every interaction with other objects kills them.

3. The existence of photons is generically connected with some time-periodical process of period T and frequency $\nu = T^{-1}$, so that the Planck relation $E = h\nu$, or $ET = h$, where h is the Planck constant, holds.

4. Every single photon carries momentum $\mathbf{p}$ with $|\mathbf{p}| = h\nu/c$ and spin momentum equal to the Planck constant h.

5. Photons are polarized objects. The polarization of every single photon should relate its spatial structure with the translational and rotational directions of propagation.

6. Photons do not interact with each other. *i.e.* they pass through each other without changes, and this allows to consider just free photons.

We recall now the basics of EED, in the frame of which the screw photon-like $3d$-solitons will be constructed.

3 Basics of extended electrodynamics in vacuum

We are going to consider just the vacuum case of EED in relativistic formulation. The signature of the space time pseudometric η is $(-,-,-,+)$, the canonical coordinates will be denoted by $(x^1, x^2, x^3, x^4) = (x, y, z, \xi = ct)$, so the volume 4-form ω_o is given by $\omega_o = \mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z \wedge \mathrm{d}\xi$. The Hodge $*$-operator is defined by $\alpha \wedge \beta = \eta(*\alpha, \beta)\omega_o$, where α and β are p and $4-p$ forms respectively. In terms of $\varepsilon_{\mu\nu\sigma\rho}$ we have $(*F)_{\mu\nu} = -\frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}F^{\sigma\rho}$. We have also the exterior derivative $\mathbf{d}$ and the coderivative $\delta = *\mathbf{d}*$. The physical interpretation of $F_{\mu\nu}$ are: $F_{i4} = -F_{4i}, i = 1, 2, 3,$ are the components $\mathbf{E}^1, \mathbf{E}^2, \mathbf{E}^3$ of the electric vector $\mathbf{E}$, and $(F_{23}, -F_{13}, F_{12})$ are the components $\mathbf{B}^1, \mathbf{B}^2, \mathbf{B}^3$ of the magnetic vector $\mathbf{B}$, respectively.

In terms of δ the vacuum Maxwell equations are given by

$$\delta * F = 0, \quad \delta F = 0. \tag{1}$$

In EED the above equations (1) are extended to

$$\delta * F \wedge F = 0, \quad \delta F \wedge *F = 0,$$
$$\delta F \wedge F - \delta * F \wedge *F = 0. \tag{2}$$

In components, equations (2) are respectively:

$$(*F)_{\mu\nu}(\delta * F)^\nu = 0, \quad F_{\mu\nu}(\delta F)^\nu = 0,$$
$$(*F)_{\mu\nu}(\delta F)^\nu + F_{\mu\nu}(\delta * F)^\nu = 0. \tag{3}$$

The physical sense of equations (3) is, obviously, local energy-momentum redistribution during the evolution: the first two equations say that F and $*F$ keep locally their energy-momentum, and the third equation says (in correspondence with the first two), that the energy-momentum transferred from F to $*F$ is always equal locally to the energy-momentum transferred from $*F$ to F, hence, any of the two expressions $F_{\mu\nu}(\delta * F)^\nu$ and $(*F)_{\mu\nu}(\delta F)^\nu$ may be considered as a measure of the rotational component of the energy-momentum redistribution between F and $*F$ during propagation (recall that the spatial part of δF contains rot$\mathbf{B}$ and the spatial part of $\delta * F$ contains rot$\mathbf{E}$).

Obviously, equations (3) have more solutions than equations (1). In particular, those solutions of (3) which satisfy the relations

$$\delta F \neq 0,\ \delta * F \neq 0 \tag{4}$$

are called *nonlinear*. Further we are going to consider only the nonlinear solutions of (3) and the corresponding "electric" and "magnetic" 3-vectors will be denoted respectively by $\mathcal{E}$ and $\mathcal{B}$.

Some of the basic results in our previous studies of the nonlinear solutions of equations (3) could be summarized in the following way: For every nonlinear solution $(F, *F)$ of (3) there exists a canonical system of coordinates (x, y, z, ξ) in which the solution is fully represented by two functions $\Phi(x, y, \xi + \varepsilon z)$, $\varepsilon = \pm 1$, and $\varphi(x, y, z, \xi)$, $|\varphi| \leq 1$, as follows:

$$\begin{aligned} F &= \varepsilon\Phi\varphi \mathrm{d}x \wedge \mathrm{d}z + \Phi\varphi \mathrm{d}x \wedge \mathrm{d}\xi \\ &\quad + \varepsilon\Phi\sqrt{1-\varphi^2}\mathrm{d}y \wedge \mathrm{d}z + \Phi\sqrt{1-\varphi^2}\mathrm{d}y \wedge \mathrm{d}\xi, \\ *F &= -\Phi\sqrt{1-\varphi^2}\mathrm{d}x \wedge \mathrm{d}z - \varepsilon\Phi\sqrt{1-\varphi^2}\mathrm{d}x \wedge \mathrm{d}\xi \\ &\quad + \Phi\varphi \mathrm{d}y \wedge \mathrm{d}z + \varepsilon\Phi\varphi \mathrm{d}y \wedge \mathrm{d}\xi. \end{aligned}$$

We call Φ the *amplitude* function and φ the *phase* function of the solution. The condition $|\varphi| \leq 1$ allows to set $\varphi = \cos\psi$, and further we are going to work with ψ, and ψ will be called *phase*. As we showed [2], the two functions Φ and φ may be introduced in a coordinate free manner, so they have well defined *invariant* sense. Every nonlinear solution satisfies the following important relations:

$$(\delta F)^2 < 0,\ (\delta{*}F)^2 < 0,\ |\delta F| = |\delta{*}F|,\ (\delta F)_\sigma(\delta{*}F)^\sigma = 0,$$
$$F_{\mu\nu}F^{\mu\nu} = F_{\mu\nu}(*F)^{\mu\nu} = 0.$$

We recall also the *scale factor* L, defined by the relation $L = |\Phi|/|\delta F|$. A simple calculation shows that it depends only on the derivatives of ψ in these coordinates and is given by

$$L = \frac{1}{|\psi_\xi - \varepsilon\psi_z|}\,. \tag{5}$$

4 Screw soliton solutions in extended electrodynamics

Note that EED considers the field as having two components: F and $*F$. As we mentioned earlier, the third equation of (3) describes how much energy-momentum is redistributed locally with time between the two components F and $*F$ of the field: $F_{\mu\nu}(\delta * F)^\nu dx^\mu$ gives the transfer from F to $*F$, and $(*F)_{\mu\nu}\delta F^\nu dx^\mu$ gives the transfer from $*F$ to F, thus, if there is such an energy-momentum exchange equations (3) require permanent and equal mutual energy-momentum transfers between F and $*F$. Since F and $*F$ are always orthogonal to each other $[F_{\mu\nu}(*F)^{\mu\nu} = 0]$ and these two mutual transfers depend on the derivatives of the field functions through δF and $\delta * F$ (*i.e.* through rot$\mathcal{B}$ and rot$\mathcal{E}$), we may interpret this property of the solution as a description of an *internal rotation-like* component of the general dynamics of the field. Hence, any of the two expressions $F_{\mu\nu}(\delta * F)^\nu dx^\mu$ or $(*F)_{\mu\nu}\delta F^\nu x^\mu$ may serve as a natural measure of this rotational component of the energy-momentum redistribution during the propagation. Therefore, after some appropriate normalization, we may interpret any of the two 3-forms $(*F) \wedge (\delta * F)$ and $F \wedge \delta F$ as local *spin-momentum* of the solution. Making use of the above expressions for F and $*F$ we compute $F \wedge \delta F = (*F) \wedge (\delta * F)$:

$$\begin{aligned} F \wedge \delta F = &-\varepsilon\Phi^2(\psi_\xi - \varepsilon\psi_z)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z \\ &- \Phi^2(\psi_\xi - \varepsilon\psi_z)\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}\xi. \end{aligned}$$

The function ψ is determined by the equation

$$\mathbf{d}(F \wedge \delta F) = 0, \tag{6}$$

which guarantees conservation of the spin-momentum. So, from (6) we obtain

$$\psi_{\xi\xi} + \psi_{zz} - 2\varepsilon\psi_{z\xi} = 0.$$

This equation has the following solutions:

1°. Running wave solutions $\psi_1 = \psi(x, y, \xi + \varepsilon z)$,

2°. $\psi_2 = \xi g(x, y, \xi + \varepsilon z) + b(x, y)$,

3°. $\psi_3 = z g(x, y, \xi + \varepsilon z) + b(x, y)$,

4°. Any linear combination of the above solutions with coefficients which are allowed to depend on (x, y). The functions $g(x, y, \xi + \varepsilon z)$ and $b(x, y)$ are arbitrary in the above expressions.

The running wave solutions ψ_1, defined by 1° lead to $F \wedge \delta F = 0$ and to $|\delta F| = 0$, and by this reason they have to be ignored. The solutions ψ_2 and ψ_3, defined respectively by 2° and 3°, give the same scale factor $L = 1/|g|$. Since at all spatial points where the field is different from zero we have $\xi + \varepsilon z =$ const., we may choose $|g(x, y, \xi + \varepsilon z)| = 1/l(x, y) > 0$, so we obtain the following *nonrunning* wave solutions of (6):

$$\psi_2 = \frac{\kappa\xi}{l(x,y)} + b(x,y); \quad \psi_3 = \frac{\kappa z}{l(x,y)} + b(x,y), \tag{7}$$

where $\kappa = \pm 1$ accounts for the two different polarizations. Clearly, the physical dimension of $l(x, y)$ is *length*, $b(x, y)$ is dimensionless and the *scale* factor is $L = l(x, y)$.

Further we consider the case 3° with $\kappa = 1$, $\varepsilon = -1$: $\kappa = 1$ means clock-wise polarization when looking along the $(+z)$-axis; $\varepsilon = -1$ means propagation along z-axis from $-\infty$ to $+\infty$. Choosing Φ appropriately, and assuming $l(x, y) = l_o =$ const.; $b(x, y) = b_o =$ const., we obtain for the electric $\mathcal{E}$ and magnetic $\mathcal{B}$ vectors the following expressions:

$$\mathcal{E} = \left\{ C\frac{\theta(x,y;r_o)\theta(\xi-z;l_o)}{\cosh\left[(x-a)^2+(y-a)^2\right]}\cos\left(\frac{z}{l_o}+b_o\right); \right. \\ \left. C\frac{\theta(x,y;r_o)\theta(\xi-z;l_o)}{\cosh\left[(x-a)^2+(y-a)^2\right]}\sin\left(\frac{z}{l_o}+b_o\right);\ 0 \right\} \quad (8)$$

$$\mathcal{B} = \left\{ -C\frac{\theta(x,y;r_o)\theta(\xi-z;l_o)}{\cosh\left[(x-a)^2+(y-a)^2\right]}\sin\left(\frac{z}{l_o}+b_o\right); \right. \\ \left. C\frac{\theta(x,y;r_o)\theta(\xi-z;l_o)}{\cosh\left[(x-a)^2+(y-a)^2\right]}\cos\left(\frac{z}{l_o}+b_o\right);\ 0 \right\} \cdot \quad (9)$$

The above expressions mean that at every moment the solution occupies one-step piece of a very *thin* screw cylinder. This screw cylinder winds around the z-axis, it's points are distant from the z-axis approximately at the same distance of l_o, so that $l_o \cong a\sqrt{2}$, and its internal radius is equal to $r_o \ll l_o$. The height of the cylinder is $2\pi l_o$, and the constant C gives the right physical dimension of the field components. We can say that this screw cylinder is made of non-intersecting screw lines. The time evolution of every point inside this screw cylinder follows its own screw line and never crosses any screw line of any other point where the field is different from zero. The two localizing functions $\theta(x,y;r_o)$ and $\theta(\xi-z)$ localize the solution at every moment ξ inside the corresponding one-step screw cylinder piece $\Omega_\xi(2\pi l_o, a, r_o)$, (for a more detailed consideration see `hep-th/0104088`).

The inverse of the scale factor $L > 0$, which was assumed to be approximately a constant l_o in this solution, characterizes how "far" is a given nonlinear solution from Maxwell solutions. In fact, when δF and $\delta * F$ go to zero, then $L \to \infty$ and $L^{-1} \to 0$.

5 The spin-momentum

According to our assumption the spin density of the solution is given by any of the correspondingly normalized two 3-forms $F \wedge \delta F$, or $(*F) \wedge (\delta * F)$. In order to have the appropriate physical dimension we consider now the 3-form β defined by

$$\beta = 2\pi\frac{L^2}{c}F\wedge\delta F = 2\pi\frac{L^2}{c}\big[-\varepsilon\Phi^2(\psi_\xi-\varepsilon\psi_z)\mathrm{d}x\wedge\mathrm{d}y\wedge\mathrm{d}z \\ -\Phi^2(\psi_\xi-\varepsilon\psi_z)\mathrm{d}x\wedge\mathrm{d}y\wedge\mathrm{d}\xi\big].$$

Its physical dimension is "energy-density $\times$ time". Since $L = L(x,y)$ at most, we see that β is closed: $\mathbf{d}\beta = 0$ and we may use the Stokes' theorem. We shall make use of the solutions 3°, so $\psi_\xi = 0, \psi_z = \kappa/l(x,y)$, $L = |\psi_\xi - \varepsilon\psi_z|^{-1}$. Assuming $L = l_o =$ const., we reduce β to the 3-space spand by (x,y,z) (we use the same notation for the reduced β)

$$\beta = \frac{2\pi l_o}{c}\kappa\Phi^2\mathrm{d}x\wedge\mathrm{d}y\wedge\mathrm{d}z,$$

integrate it over the 3-space and obtain

$$\int_{\mathcal{R}^3}\beta = \kappa E\frac{2\pi l_o}{c} = \kappa ET = \pm ET, \quad (10)$$

where E is the integral energy of the solution, $T = 2\pi l_o/c$ is the intrinsically defined time-period, and $\kappa = \pm 1$ accounts for the two polarizations. According to our interpretation this is the integral intrinsic angular momentum, or spin-momentum, of the solution, for one period T. This intrinsically defined action ET of the solution is to be identified with the Planck's constant h, $h = ET$, or $E = h\nu$, if we are going to interpret the solution as an extended model of a single photon.

6 Conclusion

We presented 3d photon-like soliton solutions with screw-like periodic dynamical structure in the frame of EED. This structure manifests its dynamical nature through the consistent rotational-translational propagation in space. The finite nature reveals itself through the finite 3-volumes of definite shape they occupy at every moment of their existence, and through the finite values of the conserved quantities they carry. The available continuous and mutual energy-momentum exchange between the two components F and $*F$ of the solution generates rotational component of its propagation being generically connected to its screw (helical) spatial structure.

References

1. J.J. Thomson, Philos. Mag. Ser. **6**, **48**, 737 (1924), and **50**, 1181 (1925), and Nature **137**, 23 (1936); N. Rashevsky, Philos. Mag. Ser. 7, **4**, 459 (1927); W. Honig, Found. Phys. **4**, 367 (1974); G. Hunter, R. Wadlinger, Phys. Essays, **2**, 158 (1989); T. Waite, Ann. de la Fondation Louis de Broglie **20**, No. 4, (1995); J.P. Vigier, Found. Phys. **21**, 125, (1991); B. Lehnert, *Photon and Old Problems in Light of New Ideas*, edited by V. Dvoeglazov (Nova Science Publ., 2000), pp. 3–20
2. S. Donev, M. Tashkova, Proc. R. Soc. Lond. A **443**, 301 (1993); S. Donev, M. Tashkova, Proc. R. Soc. Lond. A **450**, 281 (1995); S. Donev, M. Tashkova, Ann. de la Fondation Louis de Broglie **23**, No. 2, 89-97, LANL `e-print: nlin/9709006`; **23**, No. 3–4, 103 (1998), LANL `e-prints: nlin/9710001, 9710003`
3. R. Rajaraman, *Solitons and Instantons* (North Holland, Amsterdam-New York-Oxford, 1982)